ARPA ORDER NO.: 3486
8G10 Tactical Technology

12

R-2209-ARPA
March 1978

Simple Relations for the Stability of Heated Laminar Boundary Layers in Water: Modified Dunn-Lin Method

J. Aroesty, W. S. King, G. M. Harpole,
W. Matyskiela, A. R. Wazzan, C. Gazley, Jr.

A Report prepared for

DEFENSE ADVANCED RESEARCH PROJECTS AGENCY

Rand
SANTA MONICA, CA. 90406

The research described in this report was sponsored by the Defense Advanced Research Projects Agency under Contract No. MDA903-78-C-0121.

Published by The Rand Corporation

(15) MDA903-78-C-0121,
ARPA ORDER [redacted] 3486
8G10 Tactical Technology

(14) RAND /R-2209-ARPA

(11) Mar[redacted]78

Interim rept.,

(12) 28p.

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APR 24 1978
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PREFACE

Under the sponsorship of the Tactical Technology Office of the Defense Advanced Research Projects Agency, Rand has been investigating the fluid mechanics and hydrodynamics of low-drag submersible vehicles. Preliminary studies of such vehicles often involve characterization of surface heating effects on the hydrodynamic stability of laminar boundary layers in water. Elaborate numerical solutions of the Orr-Sommerfeld equations are usually required for such characterization, even for preliminary conceptual studies.

An analytic method first developed more than 30 years ago is reviewed and modified to provide a simple set of relations for determining the effect of heating and pressure gradient on the minimum critical Reynolds number of laminar boundary layers in water. This new method does not obviate the need for numerical solution of the Orr-Sommerfeld equation, such as might be required for "e^9" calculations. However, it does provide an inexpensive and simple way to estimate the effects of wall temperature and pressure gradient on the minimum critical Reynolds number.

This report should be useful to hydrodynamicists, designers of submersibles, and others engaged in the application of fluid mechanics to the improvement of underwater vehicle performance. Related Rand reports include:

- R-1752-ARPA/ONR, *Low-Speed Boundary-Layer Transition Workshop*, W. S. King, June 1975.
- R-1789-ARPA, *Controlling the Separation of Laminar Boundary Layers in Water: Heating and Suction*, J. Aroesty and S. A. Berger, September 1975.
- R-1863-ARPA, *The Effects of Wall Temperature and Suction on Laminar Boundary-Layer Stability*, W. S. King, April 1976.
- R-1898-ARPA, *"e^9": Stability Theory and Boundary-Layer Transition*, S. A. Berger and J. Aroesty, February 1977.
- R-1907-ARPA, *Buoyancy Cross-Flow Effects on the Boundary Layer of a Heated Horizontal Cylinder*, L. S. Yao and I. Catton, April 1976.

R-1966-ARPA, *The Buoyancy and Variable Viscosity Effects on a Water Laminar Boundary Layer Along a Heated Longitudinal Horizontal Cylinder*, L. S. Yao and I. Catton, February 1977.

R-2111-ARPA, *Entry Flow in a Heated Tube*, L. S. Yao, June 1977.

R-2164-ARPA, *The Effects of Unsteady Potential Flow on Heated Laminar Boundary Layers in Water: Flow Properties and Stability*, W. S. King, J. Aroesty, L. S. Yao, and W. Matyskiela, in process.

R-2165-ARPA, *Approximate Methods for Calculating the Properties of Heated Laminar Boundary Layers in Water*, G. M. Harpole, S. A. Berger, and J. Aroesty, January 1978.

SUMMARY

A simplified method is described for calculating the effect of surface heating on the hydrodynamic stability of heated laminar boundary layers in water. The method involves modification and up-dating of relations first developed by Lin for a constant-property fluid and later by Dunn and Lin for a compressible one. These modified Dunn-Lin relations for the minimum critical Reynolds number give results in substantial agreement with numerical integration of the Orr-Sommerfeld equations. The method is then employed to evaluate the influence of ambient temperature level and wall temperature distribution on the minimum critical Reynolds number. Preliminary studies indicate that wall temperature distributions can be found that have a strong favorable effect on the stability of laminar boundary layers in water, even for adverse pressure gradient flow.

CONTENTS

I. INTRODUCTION

Successful laminar flow hydrodynamics relies on the manipulation of velocity profiles in the wall region of the laminar boundary layer to enhance flow stability. Pressure gradient, body shaping, suction, and surface heat transfer are classic methods for accomplishing this manipulation.

Recent theoretical and experimental studies suggest that surface heating in water holds promise for enhancing boundary-layer stability by slowing the growth of Tollmein-Schlichting instabilities, or by increasing the surface area over which two-dimensional, infinitesimal disturbances are damped. In the absence of a comprehensive theory of boundary-layer transition, linear stability theory currently provides the sole analytic guide for manipulating mean flow velocity profiles to delay transition.

Even for preliminary engineering applications, no single parameter describes the stability of a particular velocity profile. However, the minimum critical Reynolds number can be extremely useful, both as a precise measure of the extent over which all two-dimensional, infinitesimal disturbances are damped and as a simple qualitative surrogate for the stability characteristics of a particular velocity profile [1].*

In 1946, C. C. Lin [2] published a simple set of relations for determining the minimum critical Reynolds number from the velocity profile of a constant property laminar boundary layer. The original Lin relations [2,3],

$$v(c)(1 - 2\lambda) = .58 \, , \tag{1a}$$

*Recent computations by Wazzan and Gazley [1] suggest that the minimum critical Reynolds number may also be useful in correlating transition predictions obtained by the "e^9" method.

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where

$$v(c) = \frac{-\pi \frac{\partial U}{\partial y}(0)\; U(y_c) \left(\frac{\partial^2 U}{\partial y^2}\right)(y_c)}{\left[\frac{\partial U}{\partial y}(y_c)\right]^3}, \tag{1b}$$

$$\lambda = \frac{\frac{\partial U}{\partial y}(0) \cdot y_c}{c} - 1, \tag{1c}$$

$$(Re_\delta)_{min\text{-}crit} = \frac{25 \frac{\partial U}{\partial y}(0)}{c^4}, \tag{1d}$$

y_c = height of critical layer, and c = wave speed, normalized to free-stream velocity, were based on asymptotic analysis of the Orr-Sommerfeld equations (valid for large values of αRe_δ, small values of the wave-length, α, and small values of c) and a numerical factor derived from a more complete calculational method. These relations were widely used [3] until the advent of computer-based schemes for the solution of the Orr-Sommerfeld equations. In 1946, Lees and Lin [4] extended the original Lin relations to the compressible flow of air, and later, Dunn and Lin [5] and Mack [6] presented further compressible extensions of the original Lin analysis of the neutral curve separating stable and unstable regions. However, special difficulties associated with compressibility diminished the role of this analytic approach in high-speed dynamics, and computer-based numerical solutions have since pre-empted stability analysis for air flows.

For the case of practical heated water boundary layers, the situation is simpler. Water density is nearly constant, temperature and viscosity fluctuations have little effect on stability, and the primary departure from the constant property incompressible flow originally considered by Lin is the variation of mean flow viscosity with temperature.

On the basis of numerical testing Wazzan [7,8] suggested that viscosity and temperature fluctuations are negligible and that the

appropriate Orr-Sommerfeld equation for heated water boundary layers is still fourth order but includes derivatives of mean flow viscosity with normal distance. The equation is

$$(U - c)(\phi'' - \alpha\phi) - U''\phi + \left(\frac{i}{\alpha R_\delta}\right)\Big[\mu(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi)$$
$$+ 2\mu'(\phi''' - \alpha^2\phi'') + \mu''(\phi'' + \alpha^2\phi)\Big] = 0 , \quad (2)$$

where primes refer to $d/d\eta$; $\eta = Y/\delta$; $R_\delta = U\delta/\nu_\infty$; the disturbance stream function $\psi(w,y,t) = \phi(\eta)e^{i(\alpha x - ct)}$; and α and c are real along the neutral curve. Both velocity and viscosity are normalized by their values as $\eta \to \infty$. It was also demonstrated that the omission of the terms μ' and μ'' had a small effect ($\sim 3\%$) on the numerical calculation of minimum critical Reynolds number. This is not surprising for the extremely stable flows sometimes encountered with heated water boundary layers, because the derivatives of ϕ are large in the critical region, and the terms μ' and μ'' are essentially of unit magnitude or less.

In this brief report, we show how the Lin relations and the Dunn-Lin theory may be used after slight modification to estimate the minimum critical Reynolds number for heated water boundary layers. The appropriate approximations and dimensional factors in these new relations are based on recent calculations of the Orr-Sommerfeld equations for constant property flow. The results of our modified Dunn-Lin theory are then compared with values obtained from numerical studies of the Orr-Sommerfeld equation for heated water boundary layers. Once the accuracy of these new relations has been established, they are used to indicate the effect of changing temperature levels on the minimum critical Reynolds number. The surprisingly powerful effect of temperature variation in the maintenance of a stable flow is illustrated by the case of an unfavorable pressure gradient. Because of their accuracy and simplicity, these relations can be useful in preliminary feasibility and optimization studies of laminar flow hydrodynamics.

II. REVIEW AND ANALYSIS

For convenience, we briefly review the Dunn-Lin method for solving Eq. (2) along the neutral curve (but only those aspects of the method that are instrumental in this modification; complete elaboration is found in Refs. 2, 5, or 6).

Because αR_δ is expected to be large, classic asymptotic methods should be sufficiently accurate for engineering estimates. First, the inviscid Orr-Sommerfeld equation obtained after neglecting terms of $O(1/\alpha R_\delta)$ is considered:

$$(U - c)(\phi'' - \alpha\phi) - U''\phi = 0 \ . \tag{3}$$

Following Lin [2], the solution of this equation that satisfies the outer boundary conditions $\phi(\infty) = \phi'(\infty) = 0$ is called $\phi(\eta)$ and is calculated by a power series in α^2.

The viscous solution is more complicated. The modified Tollmien variation, ξ, used for obtaining the dominant viscous solutions of Eq. (2), accounts for both departures from linearity of the velocity profile and variable viscosity. It is defined by

$$\xi = (\alpha Re_\delta)^{1/3}\left(\int_{\eta_c}^{\eta} \frac{3}{2}\sqrt{\frac{U - c}{\mu}}\, d\eta\right)^{2/3} \ . \tag{4}$$

The leading terms of Eq. (2) then become

$$F_{\zeta\zeta\zeta\zeta}(\zeta) - i\zeta\, F_{\zeta\zeta}(\zeta) \quad 0 \ , \tag{5}$$

where $F(\xi)$ is the dominant viscous solution of the Orr-Sommerfeld equation. There are four independent solutions of this equation. Two are essentially inviscid solutions and are already included in the inviscid solution $\phi(\eta)$, and one of the remaining solutions is unbounded

as $\zeta \to \infty$. Thus, there is only one acceptable viscous solution. This solution must be multiplied by a factor to give the correct asymptotic form as $\alpha R_\delta \to \infty$. This factor introduces an error not larger than $O(\alpha R_\delta)^{-1/3}$ in Eq. (5).

The appropriate viscous solution is $\phi_{vis}(\eta)$, where

$$\phi_{vis}(\eta) = -i(\alpha R_\delta)^{-1/2} \left(\frac{U - c}{\mu}\right)^{-1/2} \zeta^{1/2} A(\eta) \times \int_\infty^\zeta d\zeta \int_\infty^\zeta \zeta^{1/2} H_{1/3}^{(1)}\left[\frac{2}{3}(i\zeta)^{3/2}\right] d\zeta \tag{6a}$$

and

$$A(\eta) = \sqrt{\frac{\pi}{3}}\, e^{-(\pi i)/3} \left(\frac{\eta}{U - c}\right)^{3/4} . \tag{6b}$$

Thus the required approximate solution is the linear combination $a\phi(\eta) + b\phi_{vis}(\eta)$.

The no-slip conditions at the wall, $\phi(0) = \phi'(0) = 0$, must still be satisfied. This leads to the relations

$$a\phi(0) + b\phi_{vis}(0) = 0 , \tag{7a}$$

$$a\psi(0) + bf_{vis}(0) = 0 , \tag{7b}$$

which then result in the eigenvalue relation

$$\frac{\phi(0)}{\psi(0)} = \frac{\phi_{vis}(0)}{f_{vis}(0)} . \tag{8}$$

where $f_{vis} = i\, \phi'_{vis}(\eta)$, and

$\psi = i\, \phi'(\eta)$.

The ratio of the viscous solutions can then be written as

$$\frac{\phi_{vis}(0)}{f_{vis}(0)} = i\left(\frac{c-U}{\mu}\right)_w^{-1/2}\left(\frac{3}{2}\int_0^{\eta_c}\sqrt{\frac{c-U}{\mu}}\,d\eta\right)F(Z)\ , \tag{9a}$$

where

$$Z = -\zeta(0) = (\alpha R_\delta)^{1/3}\left[\frac{3}{2}\int_0^{\eta_c}\sqrt{\frac{c-U}{\mu}}\,d\eta\right]^{2/3}\ , \tag{9b}$$

and F(Z), the Tietjen's function, is defined by

$$F(Z) = \frac{\displaystyle\int_\infty^{-Z} d\zeta \int_\infty^{\zeta} \zeta^{1/2}\, H_{1/3}^{(1)}\left[\frac{2}{3}\,(i\zeta)^{3/2}\right] d\zeta}{\displaystyle -Z\int_\infty^{Z} \zeta^{1/2}\, H_{1/3}^{(1)}\left[\frac{2}{3}\,(i\zeta)^{3/2}\right] d\zeta}\ . \tag{9c}$$

If λ is defined by

$$\lambda \equiv \frac{U'(0)}{c}\left(\frac{c}{\mu(0)}\right)^{1/2}\left[\frac{3}{2}\left(\int_0^{\eta_c}\sqrt{\frac{c-U}{\mu}}\,d\eta\right)\right] - 1\ , \tag{10}$$

Eq. (9a) becomes

$$\frac{\phi_{vis}(0)}{f_{vis}(0)} = i(1+\lambda)\,\frac{c}{\frac{dU'(0)}{d\eta}}\,F(Z)\ , \tag{11}$$

and the eigenvalue relation (Eq. (8)) becomes

$$\frac{\phi(0)}{i\psi(0)} = \frac{c(1+\lambda)}{U'(0)}\,F(Z)\ . \tag{12}$$

It is traditional to write this equation in the form

$$E(\alpha,c) = F(Z) \ , \tag{13a}$$

where

$$E(\alpha,c) = \frac{\phi(0)}{i\psi(0)} \frac{U'(0)}{c(1 + \lambda)} \ . \tag{13b}$$

The quantity λ is usually small. If both velocity gradient and viscosity are constant, the λ is identically zero. In other more realistic cases, departures from this constancy in the region between wall and critical layer are generally small enough to result in $|\lambda| << 1$. However, its selective inclusion in numerical calculation results in more accurate estimates.

Equation (13a) can be recovered from the original Dunn-Lin formulation by setting $M_1 = 0$ and $T_w = 1$ and permitting μ to vary with η. It is also customary to introduce the modified Tietjens function defined by $\mathfrak{J}(Z) = 1/[1 - F(Z)]$, and to rewrite $E(\alpha,c)$ in terms of auxiliary real valued functions $u(\alpha,c)$ and $v(\alpha,c)$ defined by

$$\frac{1}{1 - E_{(\alpha,c)}} = \frac{(1 + \lambda)(u + iv)}{1 + \lambda(y + iv)} \ . \tag{14}$$

The secular relation (Eq. (13a)) then becomes

$$\frac{(1 + \lambda)(u + iv)}{1 + \lambda(u + iv)} = \mathfrak{J}(Z) = \mathfrak{J}_r(Z) + i\mathfrak{J}_i(Z) \ , \tag{15}$$

where $\mathfrak{J}(Z)$ is decomposed into real and imaginary components.

Lin [2] showed that the inviscid solution could be written as

$$u + iv = 1 + u'(0)c \, \frac{(1 - c)^{-2} + K_1\alpha + (1 - c)^{-2} K_2\alpha^2 + K_3\alpha^2 + \ldots}{\alpha + (1 - c)^{-2}H_1\alpha^2 + H_2\alpha^4 + H_3\alpha^6(1 - c)^{-2} + \ldots} \ , \tag{16}$$

$$\text{where } K_1(c) = \int_0^{\eta_c} (U - c)^{-2}\, dy \,,$$

$$K_2(c) = \int_0^{\eta_c} (U - c)^{-2}\, dy \int_0^{y} (U - c)^{2}\, dy \,,$$

$$K_3(c) = \int_0^{\eta_c} (U - c)^{-2}\, dy \int_0^{y} (U - c)^{2}\, dy \int_0^{y} (U - c)^{-2}\, dy \,,$$

$$H_1(c) = \int_0^{\eta_c} (U - c)^{2}\, dy \,, \tag{17}$$

$$H_2(c) = \int_0^{\eta_c} (U - c)^{2}\, dy \int_0^{y} (U - c)^{2}\, dy \,,$$

$$H_3(c) = \int_0^{\eta_c} (U - c)^{2}\, dy \int_0^{y} (U - c)^{-2}\, dy \int_0^{y} (U - c)^{2}\, dy \,.$$

By direct evaluation of Eqs. (16) and (17) for small α and c, Lin found that Eq. (1b),

$$v = -\frac{\pi U'(0)U(\eta_c)U''(\eta_c)}{\left[U'(\eta_c)\right]^3} \,,$$

is a suitable approximation for v.

Lin also determined that the minimum value of Re_δ is approximated when $\mathfrak{J}_i(Z)$ has its maximum value, corresponding to $\mathfrak{J}_i'(Z_0) = 0$, $\mathfrak{J}_i(Z_0) = .58$, $\mathfrak{J}_r(Z_0) = 1.498$, and $Z_0 = 3.21$. He then proposed an iteration

procedure, valid for small values of λ, for solving Eq. (15). The procedure is based on neglecting terms of $O(\lambda)$ in evaluating U, but retaining such terms in the evaluation of v. From Eq. (15):

$$u \cong \mho_r(Z_C) \tag{18a}$$

and

$$v\left\{1 + \lambda\left[1 - 2\mho_r(Z_0)\right]\right\} = \mho_i(Z_0) \ . \tag{18b}$$

Equation (1a), Lin's original relation for determining c, thus follows from Eq. (18b). The relation between αRe_δ and Z follows from Eqs. (9b) and (10):

$$(\alpha Re_\delta)_{critical} = \frac{(Z)^3\ U'(0)^2\ \mu(0)}{c^3(1 + \lambda)^2} \ . \tag{19}$$

Originally, Lin estimated α from a crude numerical solution of Eq. (16) for the Blasius profile,

$$\alpha \cong U'(0)c \ . \tag{20}$$

He then approximated Eq. (19) by

$$(Re_\delta)_{critical} \cong \frac{25\ U'(0)}{c^4} \tag{1d}$$

after neglecting λ and adjusting his constants to agree with results obtained from his own more detailed calculations. In the same spirit as Lin, we also assume that $\alpha = U'(0)c$, neglect terms of $0(\lambda)$ in Eq. (19), and deduce

$$(Re_{\delta *})_{critical} \cong \frac{28\ U'(0)\ \mu(0)\ \delta *}{c^4\ \delta} \ . \tag{21}$$

where the numerical constant is now chosen to agree with recent accurate numerical calculations for the constant property Blasius boundary layer.

Because λ is a small quantity, we bypass the required numerical quadrature between wall and critical layer, and with sufficient accuracy estimate it by

$$\lambda = .4\left\{\left[1 - \frac{U'(\eta_c)}{U'(0)}\right] + .5\left[1 - \frac{\mu(\eta_c)}{\mu(0)}\right]\right\} . \tag{22}$$

This formula results from the assumption of a linear viscosity profile and parabolic velocity profile between wall and critical layer and the evaluation of the dominant term in the integral in Eq. (10). The required modifications of the Dunn-Lin theory are then Eqs. (1a), (1b), (10), and (22) for determining η_c and c; and Eq. (21) for the evaluation of the minimum critical Reynolds number, $(Re_{\delta*})_{critical}$.

III. RESULTS

FIGURE 1

A comparison between values of $(Re_{\delta*})_{critical}$ calculated by the present approximate method and those calculated by numerical solution of the Orr-Sommerfeld equation is shown in Fig. 1. The lines are based on published results of Wazzan and co-workers [7-10] for Falkner-Skan similarity flows at various surface overheating. The symbols correspond to the current results. Although no single profile shape factor can uniquely correlate $(Re_{\delta*})_{critical}$ when both pressure gradient and surface heating are present, $H \equiv \delta*/\theta$ remains a convenient parameter for presenting these results.

The open circles are computed by our approximate formulas for wedge flows at zero ΔT. The agreement between exact calculation and the proposed method is remarkable in the practical range of H between 2.2 and 3.

The asterisks correspond to the case of $\beta = 0$ (flat plate) and various surface overheating. Note that the agreement between exact and approximate methods is again satisfactory in the region between $H = 2.59$ and $H \approx 2.2$, where a stability reversal occurs at ΔT increases past 75°F. The method agrees with exact calculations in predicting the existence and location of the maximum attainable critical Reynolds number, both for this case and for the case of $\beta = -.1988$ (closed circles).

For $\beta = 0$, the method appears to overpredict $(Re_{\delta*})_{critical}$ in the region of large surface overheating corresponding to values of H less than 2.3.

The case of $\beta = 1.0$ (crosses), corresponding to an extremely favorable pressure gradient, is more problematic. Exact calculations indicate that $(Re_{\delta*})_{critical}$ exhibits a local maximum at 2.2×10^4 and $H = 2.05$ and then increases again with further surface overheating. The approximate method, however, exhibits an absolute maximum at $H = 2.05$, corresponding to $\Delta T \approx 40°F$. The agreement between the exact and approximate values is good up to this value of surface overheating.

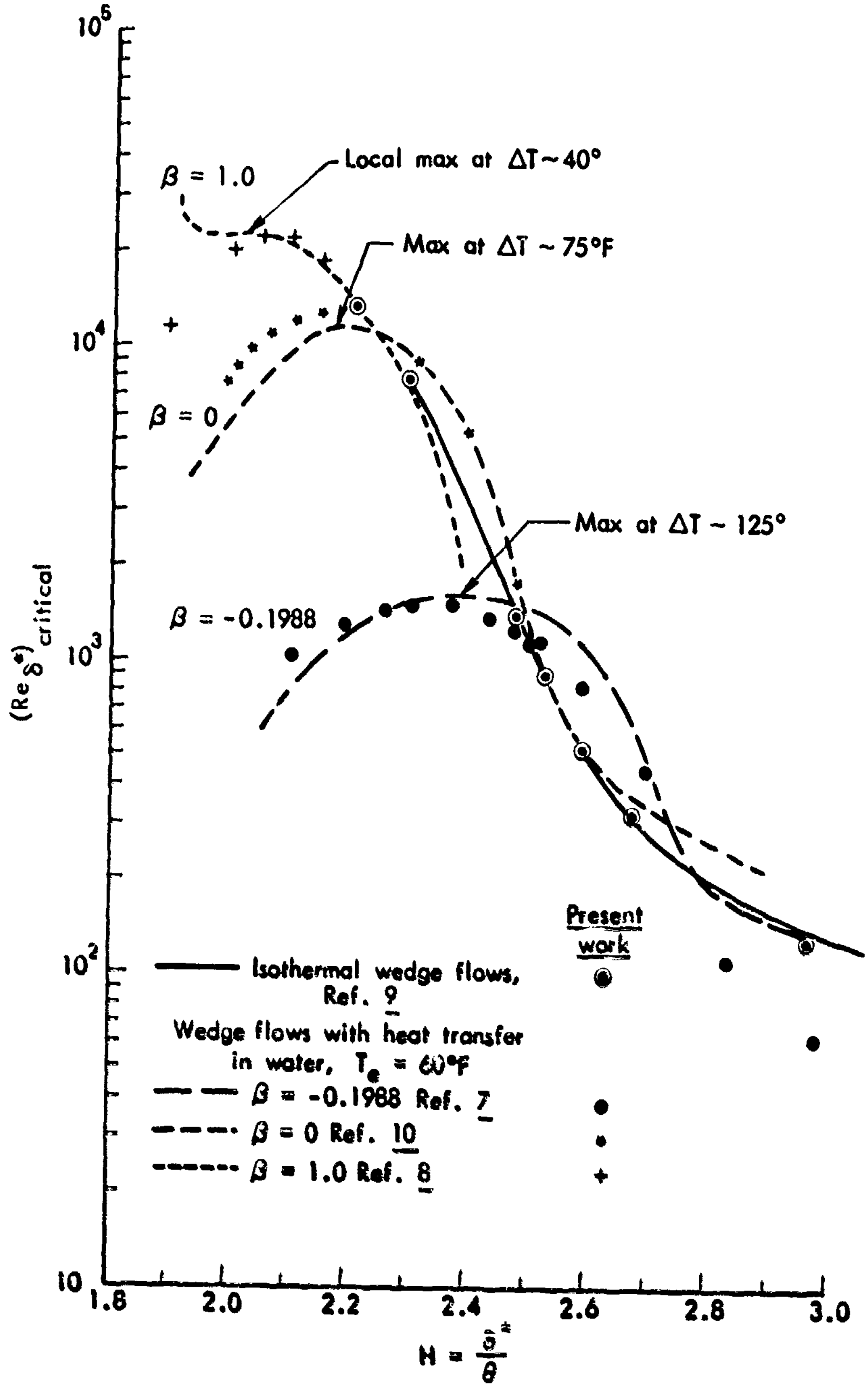

Fig. 1—Comparison between modified Dunn-Lin method and numerical solution of the Orr-Sommerfeld equation

The modified Dunn-Lin method can be used with reasonable confidence when moderate surface overheating, of the type achievable in practice, is considered.

FIGURE 2

Figure 2 shows the variation of $(Re_x)_{critical}$ with surface overheat for a series of favorable and unfavorable pressure gradients of the Falkner-Skan type. The symbols in this figure represent results obtained by numerical integration of the Orr-Sommerfeld equations, and the lines refer to the approximate method.

The agreement between exact and approximate computations is again satisfactory in the practical range of "βs" between +.10 and -.10. These results support our previous observation about the method being accurate for practical levels of pressure gradient and surface overheating.

FIGURE 3

In Fig. 3, the modified Dunn-Lin method is used to calculate the dependence of $(Re_{\delta*})_{critical}$ on ambient water temperature for several surface overheats and βs of .10, 0, and -.10. The variation of water viscosity with temperature is the primary mechanism for the surprisingly large effect of ambient water temperature on $(Re_{\delta*})_{critical}$, corresponding to a three-fold increase when ambient temperature is decreased from 75°F to 45°F.

FIGURE 4

All of the previous results are for constant wall temperature. Figure 4 compares results obtained by the present method with exact computations for variable wall temperature ($\Delta T \sim X$) and an adverse pressure gradient corresponding to $\beta = -.10$. The ability of the approximate method to predict the qualitative features of the $(Re_{\delta*})_{critical}$ variation with ΔT is again demonstrated. Note that the variable wall temperature increases stability over the constant wall temperature case when the local temperature difference is greater than 45°F. The surprisingly powerful effect of wall temperature

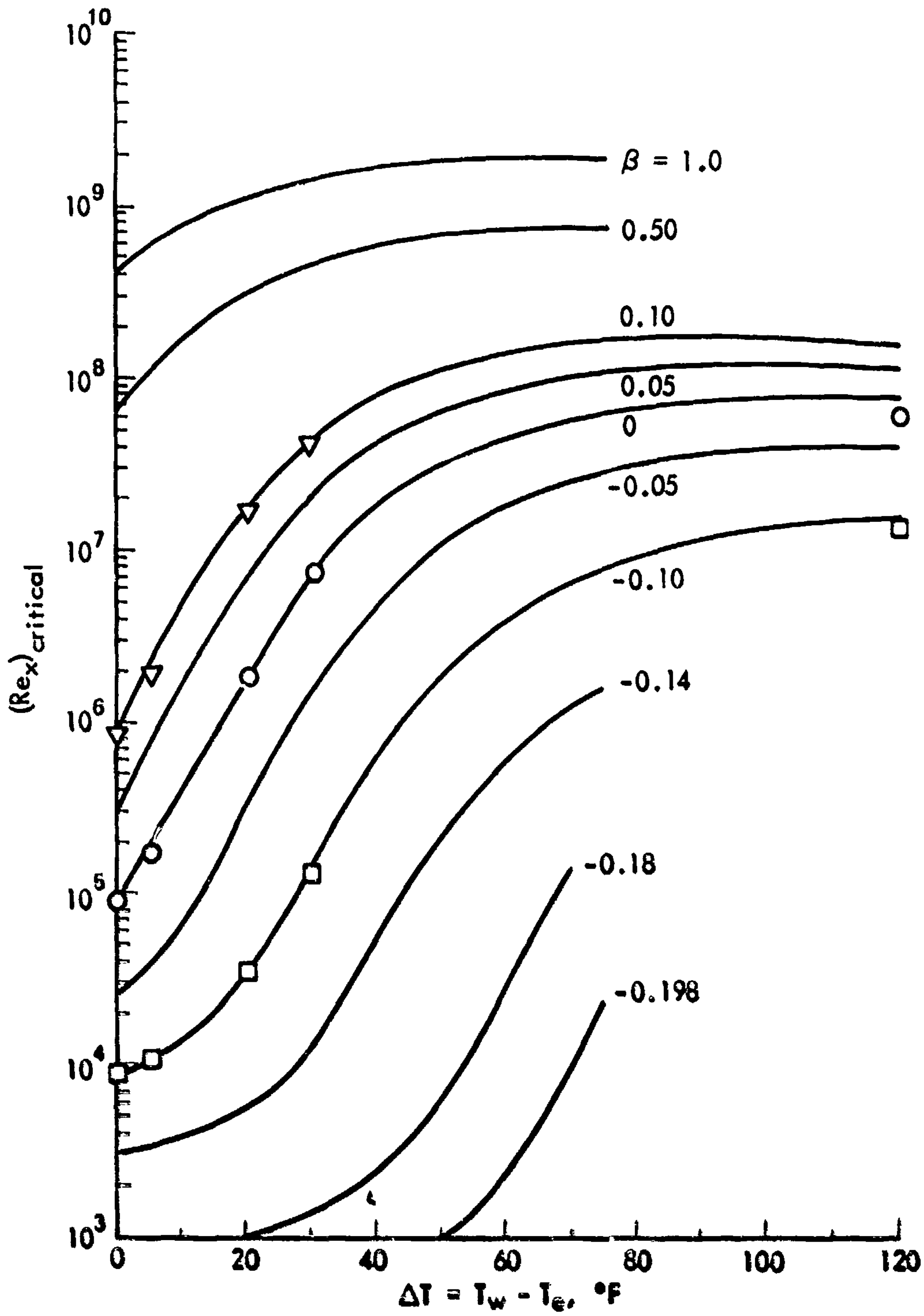

Fig. 2 — The minimum critical Reynolds number for laminar water boundary layers (T_e = 67°F) computed by the modified Dunn-Lin approximation. Symbols are exact computations.

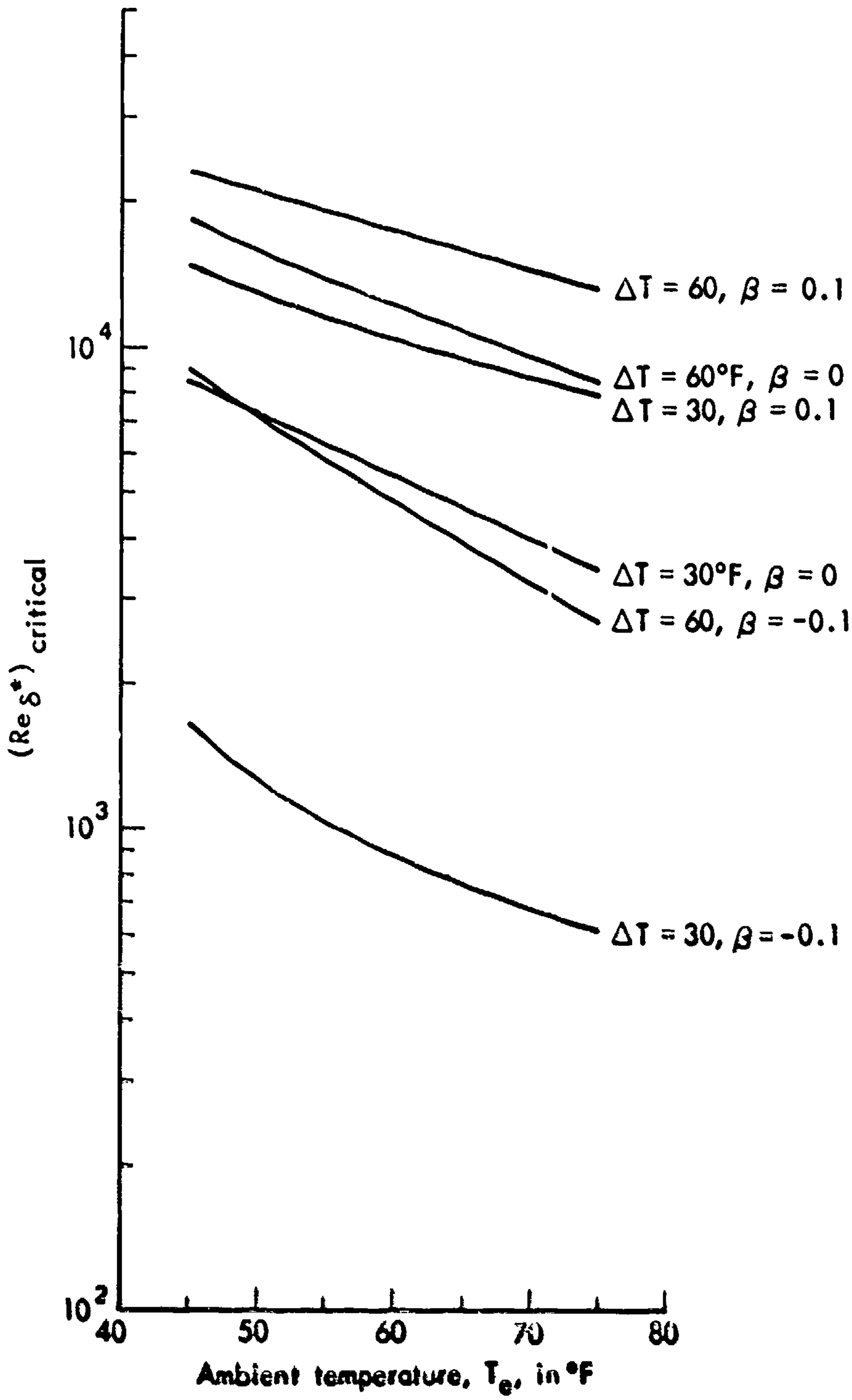

Fig. 3 — The effect of ambient temperature on minimum critical Reynolds number

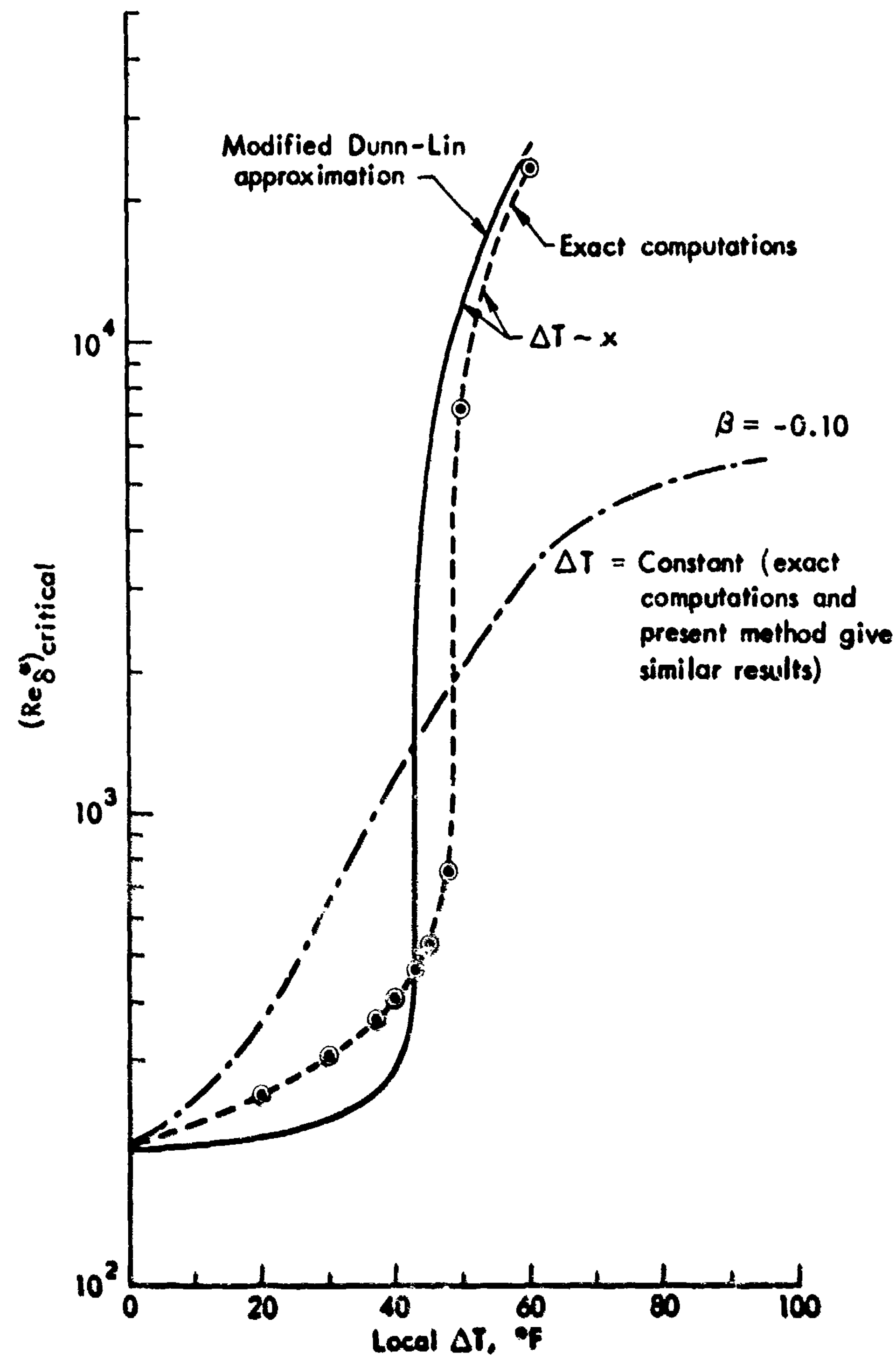

Fig. 4—The effect of variable surface overheat on neutral stability: Modified Dunn-Lin vs. exact computations (T_e = 67°)
(Note the increase in stability over the constant wall temperature case when ΔT > 45°F)

variation in increasing the extent of stable laminar flow is due primarily to the upstream history of the thermal boundary layer and its ability to appropriately deform viscosity and velocity profiles in the region between critical layer and wall.

CONCLUSION

We have developed a modification of Lin's method that can be used to estimate the minimum critical Reynolds number of a heated laminar boundary layer. This method is both simple and surprisingly accurate. As in Lin's original work, the method is most applicable when $c \to 0$, $\alpha \to 0$, and $\alpha Re_\delta \to \infty$. The internal details of the method do not agree entirely with exact analyses in other ranges of these parameters, but the few required constants have been chosen so that predictions of minimum critical Reynolds number are consistent with results of more elaborate numerical integration of the Orr-Sommerfeld equations.

This method, which we have labeled the "modified Dunn-Lin method," can be used with confidence in engineering studies of laminar boundary layer control in water. Preliminary studies, based on this method, suggest that an appropriate surface temperature distribution can promote stability, even for flows experiencing adverse pressure gradients.

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4. TITLE (and Subtitle) Simple Relations for the Stability of Heated Laminar Boundary Layers in Water: Modified Dunn-Lin Method		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. Aroesty, W. S. King, G. M. Harpole, W. Matyskiela, A. R. Wazzan, C. Gazley, Jr.		8. CONTRACT OR GRANT NUMBER(s) MDA903-78-C-0121
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Rand Corporation 1700 Main Street Santa Monica, California 90406		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE March 1978
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Laminar Boundary Layer
Hydrodynamics
Fluid Mechanics
Submarines
Underwater Vehicles

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

see reverse side

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

A simplified method for calculating the effect of surface heating on the hydrodynamic stability of heated laminar boundary layers in water. The method involves modification and updating of relations first developed by Lin for a constant property fluid and later by Dunn and Lin for a compressible one. These modified Dunn-Lin relations for the minimum critical Reynolds number give results in substantial agreement with numerical integration of the Orr-Sommerfeld equations. The powerful effects of surface temperature distribution on the stability of laminar boundary layers in water is illustrated, even for adverse pressure gradient.
Ref. (author)